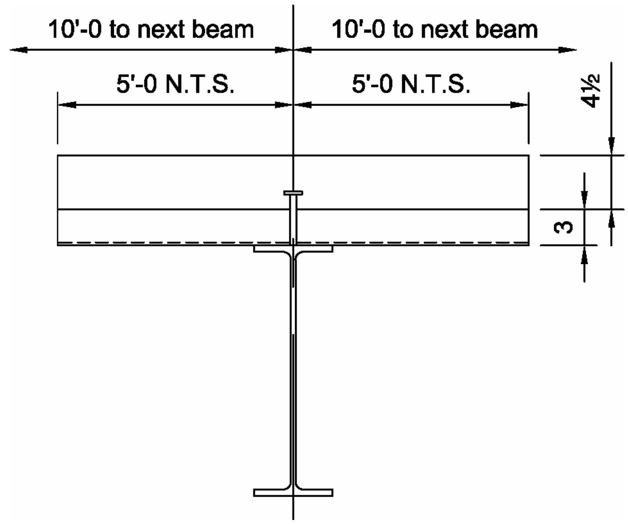
Example I-1 Composite Beam Design

Given:

A series of 45-ft. span composite beams at 10 ft. o/c are carrying the loads shown below. The beams are ASTM A992 and are unshored. The concrete has f'_c = 4 ksi. Design a typical floor beam with 3 in. 18 gage composite deck, and 4½ in. normal weight concrete above the deck, for fire protection and mass. Select an appropriate beam and determine the required number of shear studs.



Solution:

Material Properties:

Concrete f'_c = 4 ksi

Beam F_y = 50 ksi F_u = 65 ksi — Manual Table 2-3

Loads:

Dead load:
- Slab = 0.075 kip/ft^2
- Beam weight = 0.008 kip/ft^2 (assumed)
- Miscellaneous = 0.010 kip/ft^2 (ceiling etc.)

Live load:
- Non-reduced = 0.10 kips/ft^2

Since each beam is spaced at 10 ft. o.c.

Total dead load = 0.093 kip/ft^2(10 ft.) = 0.93 kips/ft.
Total live load = 0.10 kip/ft^2(10ft.) = 1.00 kips/ft.

Construction dead load (unshored) = 0.083 kip/ft^2(10 ft) = 0.83 kips/ft
Construction live load (unshored) = 0.020 kip/ft^2(10 ft) = 0.20 kips/ft

Determine the required flexural strength

LRFD	ASD
w_u = 1.2(0.93 kip/ft) + 1.6(1.0 kip/ft) = 2.72 kip/ft $M_u = \dfrac{2.72 \text{ kip/ft}(45 \text{ ft})^2}{8}$ = 687 kip-ft.	w_a = 0.93 kip/ft + 1.0 kip/ft = 1.93 kip/ft $M_a = \dfrac{1.93 \text{ kip/ft}(45 \text{ ft})^2}{8}$ = 489 kip-ft.

Use Tables 3-19, 3-20 and 3-21 from the Manual to select an appropriate member

Determine b_{eff} — Section I.3.1.1a

The effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

(1) one-eighth of the beam span, center to center of supports

$$\frac{45 \text{ ft.}}{8}(2) = 11.3 \text{ ft.}$$

(2) one-half the distance to center-line of the adjacent beam

$$\frac{10 \text{ ft.}}{2}(2) = 10.0 \text{ ft.}$$ **Controls**

(3) the distance to the edge of the slab
Not applicable

*Calculate the moment arm for the concrete force measured from the top of the steel shape, Y*2.

Assume a = 1.0 in. (Some assumption must be made to start the design process. An assumption of 1.0 in. has proven to be a reasonable starting point in many design problems.)

$Y2 = t_{slab} - a/2$ = 7.5 – ½ = 7.0 in.

Enter Manual Table 3-19 with the required strength and $Y2$=7.0 in. Select a beam and neutral axis location that indicates sufficient available strength. — Manual Table 3-19

Select a W21×50 as a trial beam.

When PNA location 5 (BFL), this composite shape has an available strength of:

LRFD	ASD
$\phi_b M_n$= 770 kip-ft > 687 kip-ft **o.k.**	M_n/Ω_b = 512 kip-ft > 489 kip-ft **o.k.**

Manual Table 3-19

Note that the required PNA location for ASD and LRFD differ. This is because the live to dead load ratio in this example is not equal to 3. Thus, the PNA location requiring the most shear transfer is selected to be acceptable for ASD. It will be conservative for LRFD.

Check the beam deflections and available strength

Check the deflection of the beam under construction, considering only the weight of concrete as contributing to the construction dead load.

Limit deflection to a maximum of 2.5 in. to facilitate concrete placement.

$$I_{req} = \frac{5}{384}\frac{w_{DL}l^4}{E\Delta} = \frac{5(0.83 \text{ kip/ft})(45 \text{ ft})^4(1728 \text{ in.}^3/\text{ft}^3)}{384(29{,}000 \text{ ksi})(2.5 \text{ in.})} = 1{,}060 \text{ in.}^4$$

From Manual Table 3-20, a W21×50 has I_x = 984 in.^{4}, therefore this member does not satisfy the deflection criteria under construction.

Using Manual Table 3-20, revise the trial member selection to a W21×55, which has I_x = 1140 in.^{4}, as noted in parenthesis below the shape designation.

Check selected member strength as an un-shored beam under construction loads assuming adequate lateral bracing through the deck attachment to the beam flange.

LRFD	ASD
Calculate the required strength 1.4 *DL* = 1.4 (0.83 kips/ft) = 1.16 kips/ft 1.2*DL*+1.6*LL* = 1.2 (0.83) + 1.6(0.20) = 1.32 klf $M_{u\text{ (unshored)}} = \dfrac{1.31 \text{ kip/ft}(45 \text{ ft})^2}{8}$ =331 kip-ft The design strength for a W21×55 is 473 kip-ft > 331 kip-ft **o.k.**	*Calculate the required strength* *DL*+*LL* = 0.83 + 0.20 = 1.03 kips.ft $M_{a\text{ (unshored)}} = \dfrac{1.03 \text{ kips/ft}(45 \text{ ft})^2}{8}$ = 260 kip-ft The allowable strength for a W21× 55 is 314 kip-ft > 260 kip-ft **o.k.**

For a W21×55 with $Y2$=7.0 in, the member has sufficient available strength when the PNA is at location 6 and $\sum Q_n$ = 292 kips. Manual Table 3-19

LRFD	ASD
$\phi_b M_n$=767 kip-ft > 687 kip-ft **o.k.**	M_n/Ω_b = 510 kip-ft > 489 kip-ft **o.k.**

Manual Table 3-19

Check a

$$a = \frac{\sum Q_n}{0.85 f_c' b} = \frac{292 \text{ kips}}{0.85(4 \text{ ksi})(10 \text{ ft.})(12 \text{ in./ft.})} = 0.716 \text{ in.}$$

0.716 in. < 1.0 in. assumed **o.k.**

Check live load deflection

$\Delta_{LL} < l/360$ = ((45 ft.)(12 in./ft))/360 = 1.5 in.

A lower bound moment of inertia for composite beams is tabulated in Manual Table 3-20.

For a W21×55 with $Y2$=7.0 and the PNA at location 6, I_{LB} = 2440 in.^{4} — Manual Table 3-20

$$\Delta_{LL} = \frac{5}{384}\frac{w_{LL}l^4}{EI_{LB}} = \frac{5(1.0\text{ kip/ft})(45\text{ ft})^4(1728\text{ in.}^3/\text{ft}^3)}{384(29{,}000\text{ ksi})(2440\text{ in.}^4)} = 1.30\text{ in.}$$

1.30 in. < 1.5 in.   **o.k.**

Determine if the beam has sufficient available shear strength

LRFD	ASD
$V_u = \frac{45\text{ft}}{2}(2.72\text{kip/ft})$ = 61.2 kips ϕV_n= 234 kips > 61.2 kips **o.k.**	$V_a = \frac{45\text{ft}}{2}(1.93\text{kip/ft})$ = 43.4 kips V_n/Ω = 156 kips > 43.4 kips **o.k.**

Manual Table 3-3

Determine the required number of shear stud connectors

Using perpendicular deck with one ¾-in. diameter weak stud per rib in normal weight 4 ksi concrete. Q_n = 17.2 kips/stud — Manual Table 3-21

$$\frac{\sum Q_n}{Q_n} = \frac{292\text{ kips}}{17.2\text{ kips}} = 17\text{, on each side of the beam.}$$

Section 3.2d(5)

Total number of shear connectors; use 2(17) = 34 shear connectors. — Section 3.2d(6)

Check the spacing of shear connectors

Since each flute is 12 in., use one stud every flute, starting at each support, and proceed for 17 studs on each end of the span.

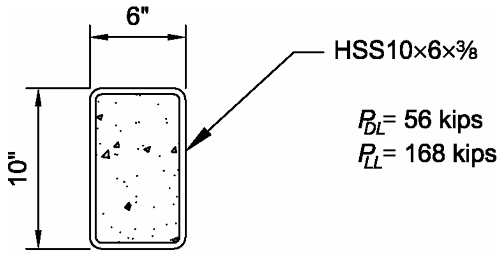
$6d_{stud}$ < 12 in. < $8t_{slab}$, therefore, the shear stud spacing requirements are met. — Section I3.2c (b)

The studs are to be 5 in. long, so that they will extend a minimum of 1½ in. into slab.

Example I-2 Filled Composite Column in Axial Compression

Given:

Determine if a 14-ft long HSS10×6×⅜ ASTM A500 grade B column filled with f'_c = 5 ksi normal weight concrete can support a dead load of 56 kips and a live load of 168 kips in axial compression. The column is pinned at both ends and the concrete at the base bears directly on the base plate. At the top, the load is transferred to the concrete in direct bearing.



Solution:

Calculate the required compressive strength

LRFD	ASD
P_u = 1.2(56 kips) + 1.6(168 kips) = 336 kips	P_a = 56 kips + 168 kips = 224 kips

The available strength in axial compression can be determined directly from the Manual at KL = 14 ft as:

LRFD	ASD
$\phi_c P_n$ = 353 kips 353 kips > 336 kips **o.k.**	P_n/Ω_c = 236 kips 236 kips > 224 kips **o.k.**

Manual Table 4-14

Supporting Calculations

The available strength of this filled composite section can be most easily determined by using Table 4-14 of the Manual. Alternatively, the available strength can be determined by direct application of the Specification requirements, as illustrated below.

Material Properties:

HSS10×6×3/8 F_y= 46 ksi F_u = 58 ksi

Manual Table 2-3

Concrete f'_c = 5 ksi $E_c = w^{1.5}\sqrt{f'_c} = (145)^{1.5}\sqrt{5} = 3{,}900$ ksi

Geometric Properties:

HSS10×6×3/8 t = 0.375 in. b = 10.0 in. h = 6.0 in. Manual Table 1-11

Concrete:
The concrete area is calculated as follows

$r = 2t = 2(0.375 \text{ in.}) = 0.75$ in. (outside radius)
$b_f = b\text{-}2r = 10 - 2(0.75) = 8.50$ in.
$h_f = h\text{-}2r = 6 - 2(0.75) = 4.50$ in.

$$A_c = b_f h_f + \pi(r\text{-}t)^2 + 2b_f(r\text{-}t) + 2h_f(r\text{-}t)$$
$$= (8.50 \text{ in.})(4.50 \text{ in.}) + \pi(0.375 \text{ in.})^2 + 2(8.50 \text{ in.})(0.375 \text{ in.}) + 2(4.50 \text{ in.})(0.375 \text{ in.})$$
$$= 48.4 \text{ in.}^2$$

$$I_c = \frac{b_1 h_1^3}{12} + \frac{2(b_2)(h_2)^3}{12} + 2(r-t)\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) + 2\left(\frac{\pi(r-t)^2}{2}\right)\left(\frac{h_2}{2} + \frac{4(r-t)}{3\pi}\right)^2$$

For this shape, buckling will take place about the weak axis, thus

$$h_1 = 6 - 2(0.375) = 5.25 \text{ in.}$$
$$b_1 = 10 - 4(0.375) = 8.5 \text{ in.}$$
$$h_2 = 6 - 4(0.375) = 4.5 \text{ in.}$$
$$b_2 = 0.375 \text{ in.}$$
$$(r-t) = 0.75 - 0.375 = 0.375 \text{ in.}$$

$$I_c = \frac{(8.5)(5.25)^3}{12} + \frac{2(0.375)(4.50)^3}{12} + 2(0.375)^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$$
$$+ 2\left(\frac{\pi(0.375)^2}{2}\right)\left(\frac{4.5}{2} + \frac{4(0.375)}{3\pi}\right)^2$$
$$= 111 \text{ in.}^4$$

Manual Table 1-11

HSS10×6×⅜:
A_s = 10.4 in.^{2} I_s = 61.8 in.^{4} h/t = 25.7

Limitations: Section I1.2

1) Normal weight concrete 10 ksi ≥ f'_c ≥ 3 ksi f'_c = 5 ksi **o.k.**

2) Not Applicable.

3) The cross-sectional area of the steel HSS shall comprise at least one percent of the total composite cross section. Section I2.2a

   10.4 in.^{2} > (0.01)(48.6 in.^{2}+ 10.4 in.^{2}) = 0.590 in.^{2} **o.k.**

4) The maximum b/t ratio for a rectangular HSS used as a composite column shall be equal to $2.26\sqrt{E/F_y}$.

   $b/t = 25.7 \le 2.26\sqrt{E/F_y} = 2.26\sqrt{29{,}000\text{ksi}/46\text{ksi}} = 56.7$ **o.k.**

   User note: For all rectangular HSS sections found in the Manual the *b/t* ratios do not exceed $2.26\sqrt{E/F_y}$.

5) Not Applicable.

Calculate the available compressive strength

C_2 = 0.85 for rectangular sections — Sec. I2.2b

$P_o = A_sF_y + A_{sr}F_{yr} + C_2A_cf'_c$ — Eqn. I2-13
$\quad = (10.4 \text{ in.}^2)(46 \text{ ksi}) + 0.85(48.4 \text{ in.}^2)(5 \text{ ksi}) = 684 \text{ kips}$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) = 0.6 + 2\left(\frac{10.4\text{in.}^2}{48.4\text{in.}^2 + 10.4\text{in.}^2}\right) = 0.954 \geq 0.90$$ Eqn. I2-15

Therefore use 0.90

$EI_{eff} = E_sI_s + E_sI_{sr} + C_3E_cI_c$ — Eqn. I2-14
$\quad = (29{,}000 \text{ ksi})(61.8 \text{ in.}^4) + (0.90)(3{,}900 \text{ ksi})(111 \text{ in.}^4)$
$\quad = 2{,}180{,}000 \text{ kip-in.}^2$

User note: K value is from Chapter C and for this case K = 1.0.

$P_e = \pi^2(EI_{eff})/(KL)^2 = \pi^2(2{,}180{,}000 \text{ kip-in.}^2) / ((1.0)(14\text{ft})(12\text{in./ft}))^2 = 762 \text{ kips}$

Eqn. I2-4

$$\frac{P_o}{P_e} = \frac{684 \text{ kips}}{762 \text{ kips}} = 0.898$$

$0.898 \leq 2.25$ Therefore use Eqn. I2-2 to solve P_n

Section I2.1b

$$P_n = P_o\left[0.658^{\left(\frac{P_o}{P_e}\right)}\right] = (684 \text{ kips})\left[0.658^{(0.898)}\right] = 470 \text{ kips}$$

Eqn. I2-6

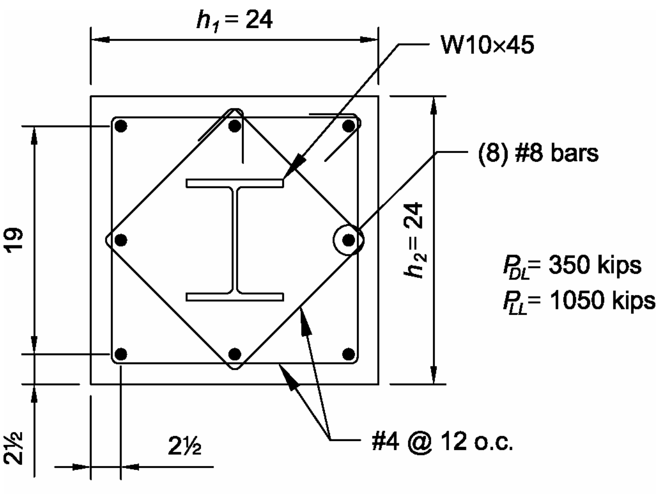
LRFD	ASD
$\phi_c = 0.75$	$\Omega_c = 2.00$
$\phi_cP_n = 0.75(470\text{kips}) = 353 \text{ kips}$	$P_n/\Omega_c = 470 \text{ kips} / 2.00 = 235 \text{ kips}$
353 kips > 336 kips **o.k.**	235 kips > 224 kips **o.k.**

Section I2.1b

Example I-3 Encased Composite Column in Axial Compression

Given:

Determine if a 14 ft tall W10×45 steel section encased in a 24 in.×24in. concrete column with f'_c = 5 ksi, is adequate to support a dead load of 350 kips and a live load of 1050 kips in axial compression. The concrete section has 8-#8 longitudinal reinforcing bars and #4 transverse ties @ 12in. o/c., The column is pinned at both ends and the load is applied directly to the concrete encasement.



Solution:

Material Properties:

Column W10×45	F_y = 50 ksi	F_u = 65 ksi	Manual Table 2-3
Concrete	f'_c = 5 ksi	E_c = 3,900 ksi (145 pcf concrete)	
Reinforcement	F_{yst} = 60 ksi		

Geometric Properties:

W10×45: (Manual Table 1-1)
A_s = 13.3 in.^{2}
I_y = 53.4 in.^{4}

Reinforcing steel:
A_{sr} = 6.32 in.^{2} (the area of 1-#8 bar is 0.79 in.^{2}, per ACI)

$$I_{sr} = \frac{\pi r^4}{4} + Ad^2 = 8\frac{\pi(0.50)^4}{4} + 6(0.79)(9.5)^2 = 428 \text{ in.}^4$$

Concrete:
$A_c = A_{cg} - A_s - A_{sr}$ = 576 in.^{2}- 13.3 in.^{2} - 6.32 in.^{2} = 556 in.^{2}
$I_c = I_{cg} - I_s - I_{sr}$ = 27,200 in.^{4}

Note: The weak axis moment of inertia is used as part of a slenderness check.

Limitations:

1) Normal weight concrete 10 ksi $\geq f'_c \geq$ 3 ksi f'_c = 5 ksi   **o.k.**   Section I1.2

2) $F_{yst} \leq$ 75 ksi F_{yst} = 60 ksi   **o.k.**

3) The cross-sectional area of the steel core shall comprise at least one percent of the total composite cross section.   Section I2.1a

   13.3 in.^{2} > (0.01)(576 in.^{2}) = 5.76in.^{2} **o.k.**

4) Concrete encasement of the steel core shall be reinforced with continuous longitudinal bars and lateral ties or spirals. The minimum transverse reinforcement shall be at least 0.009 in.^{2} of tie spacing.

   0.20 in.^{2}/12 in. = 0.0167 in.^{2}/in. > 0.009 in.^{2}/in.   **o.k.**

5) The minimum reinforcement ratio for continuous longitudinal reinforcing, ρ_{sr}, shall be 0.004.

$$\rho_{sr} = \frac{A_{sr}}{A_g} = \frac{6.32\text{in.}^2}{576\text{in.}^2} = 0.011 > 0.004 \quad \textbf{o.k.}$$

Eqn. I2-1

Calculate the total required compressive strength

LRFD	ASD
P_u = 1.2(350 kips) + 1.6(1050 kips) = 2100 kips	P_a = 350 kips + 1050 kips = 1400 kips

Calculate the available compressive strength

$P_o = A_sF_y + A_{sr}F_{yr} + 0.85A_c f'_c$
= (13.3 in.^{2})(50 ksi) + (6.32 in.^{2})(60 ksi) +0.85(556 in.^{2})(5 ksi) = 3410 kips   Eqn. I2-2

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) = 0.1 + 2\left(\frac{13.3\text{ in.}^2}{556\text{ in.}^2 + 13.3\text{ in.}^2}\right) = 0.15$$

Eqn. I2-5

$EI_{eff} = E_sI_s + 0.5E_sI_{sr} + C_1E_cI_c$ Eqn. I2-4
= (29,000 ksi)(53.4 in.^{4}) + 0.5(29,000 ksi)(428 in.^{4}) + (0.15)(3,900 in.^{4})(27,200)
=23,700,000 kip-in.^{2}

User note: K value is from Chapter C and for this case K = 1.0.

$P_e = \pi^2(EI_{eff})/(KL)^2 = \pi^2$(23,700,000 kip-in.^{2}) / ((1.0)(14in.)(12in./ft))^2 = 8,290 kips

Eqn. I2-3

$$\frac{P_o}{P_e} = \frac{3,410\text{ kips}}{8,290\text{ kips}} = 0.411$$

0.411 $\leq$ 2.25   Therefore use Eqn. I2-2 to solve P_n Section I2.1b

Eqn. I2-6

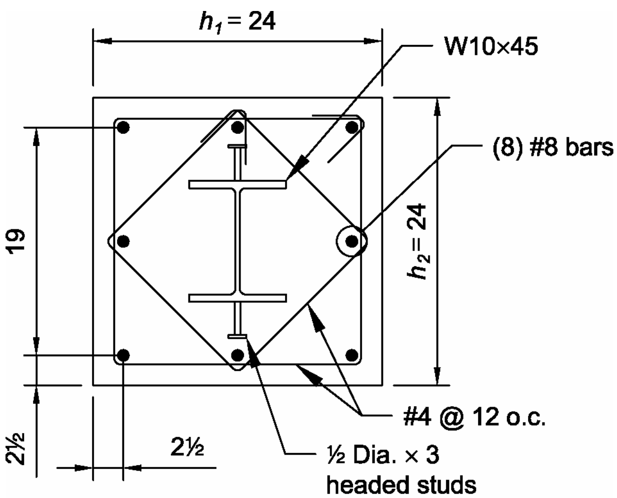
$$P_n = P_o\left[0.658^{\left(\frac{P_o}{P_e}\right)}\right] = (3{,}410\text{ kips})\left[0.658^{(0.411)}\right] = 2870\text{ kips}$$

Section I2.1b

LRFD	ASD
$\phi_c = 0.75$	$\Omega_c = 2.00$
$\phi_c P_n = 0.75(2870\text{kips}) = 2150$ kips	$P_n/\Omega_c = 2870$ kips / 2.00 =1440 kips
2150 kips > 2100 kips **o.k.**	1440 kips > 1400 kips **o.k.**

Because the entire load in the column was applied directly to the concrete, accommodations must be made to transfer an appropriate portion of the axial force to the steel column. This force is transferred as a shear force at the interface between the two materials.

Determine the number and spacing of ½-in. diameter headed shear studs to transfer the axial force.



Solution:

Material Properties:

Conc. $f'_c = 5$ ksi $E_c = 4070$ ksi

Shear Studs $F_u = 65$ ksi

Manual Table 3-21

Geometric Properties:

W10×45 $A_{st} = 13.3$ in.^{2} $A_{flange} = 4.97$ in.^{2} $A_{web} = 3.36$ in.^{2} $d_{st} = 10.1$ in.
Shear Studs $A_{sc} = 0.196$ in.^{2}
Conc. $d_c = 24$ in.

Manual Table 1-1

Calculate the shear force to be transferred

LRFD	ASD
$V = \dfrac{P_u}{\phi_c} = \dfrac{2{,}100}{0.75} = 2{,}800$ kips	$V = P_a\Omega_c = 1{,}400(2) = 2{,}800$ kips

$V' = V(A_sF_y/P_o)$ = 2800 kips ((13.3 in.^{2})(50 ksi) / (3410 kips)) = 546 kips

Eqn. I2-10

Calculate the nominal strength of one ½ in. diameter shear stud connector

$Q_n = 0.5A_{sc}\sqrt{f_c^{'}E_c} \leq A_{sc}F_u$

$0.5A_{sc}\sqrt{f_c^{'}E_c} = 0.5(0.196 \text{ in.}^2)\sqrt{(5\text{ksi})(3900\text{ksi})} = 13.7$ kips

$A_{sc}F_u = (0.196 \text{ in.}^2)(65 \text{ ksi}) = 12.7$ kips

Eqn. I2-12

Therefore use 12.7 kips.

Calculate the number of shear studs required to transfer the total force, V'

$V' / Q_n = 546$ kips / 12.7 kips = 43

An even number of studs are required to be placed symmetrically on two faces. Therefore use 22 studs minimum per flange

Determine the spacing for the shear studs

Section I2.1f

The maximum stud spacing is 16 in.

The available column length is 14ft (12 in./ft) = 168 in. and the maximum spacing is = 168 in./(22+1) = 7.3 in.

Therefore, on the flanges, use single studs @ 7 in.

Stud placement is to start 10.5 in. from one end.

Determine the length of the studs for the flanges;

$$\left(\frac{d_c - d_{st}}{2}\right) - 3 \text{ in.} = \left(\frac{24 \text{ in.} - 10.1 \text{ in.}}{2}\right) - 3 \text{ in.} = 3.95 \text{ in.}$$

Therefore use 3 ½ in. in length for the flanges.

Note: The subtraction of 3 in. is to ensure sufficient cover.

Summary: use ½-in. diameter shear stud connectors as shown on each flange, spaced @ 7 in.