# ENCE 455
# Design of Steel Structures

## V. Beam-Columns

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## Beam-Column

Following subjects are covered:

- Interaction formulas
- Moment amplification
- Stability: Braced vs. unbraced frames
- Design of beam-column
- Trusses with top-chord loads

Reading:

- Chapters 6 of Segui
- AISC Steel Manual Specification Chapters C (Stability Analysis and Design) and Appendix 1 (Inelastic Analysis and Design), 6 (Stability Bracing for Columns and Beams) and 7 (Direct Analysis Method)



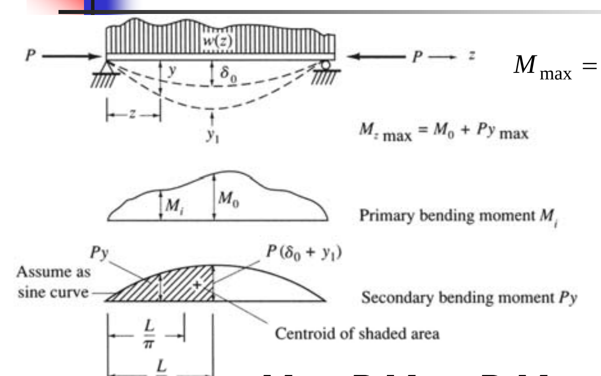
## Interaction Formulas

- Doubly and Singly Symmetric Members in Flexure and Compression
  - $For\ \frac{P_u}{\phi_c P_n} \geq 0.2$ $\quad \frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$ (H1-1a)
  - $For\ \frac{P_u}{\phi_c P_n} < 0.2$ $\quad \frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$ (H1-1b)
- Unsymmetric and Other Members in Flexure and Compression
  - $\left|\frac{f_a}{F_a} + \frac{f_{bw}}{F_{bw}} + \frac{f_{bz}}{F_{bz}}\right| \leq 1.0$ (H2-1)

(Segui Example 6.1)



## Moment Amplification




$$M_{max} = P(y_0 + y) = M_0 / [1 - (P/P_e)]$$

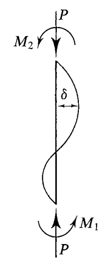
$$AmpFactor = \frac{1}{1-(P/P_e)}$$

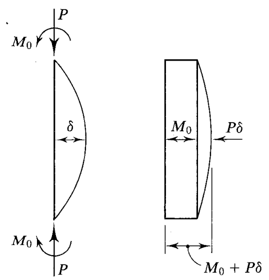
$$M_r = B_1 M_{nt} + B_2 M_{lt}$$

$B_1$, $M_{nt}$: Amplification factor & maximum moment for braced frame
$B_2$, $M_{lt}$: Amplification factor & maximum moment for unbraced frame

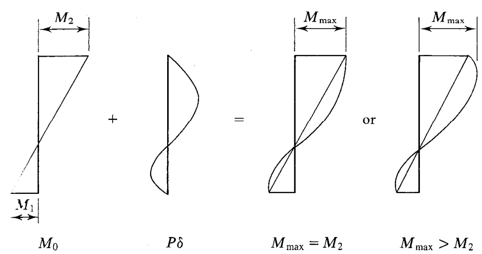
## Braced Frames

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1$$





Single-curvature bending



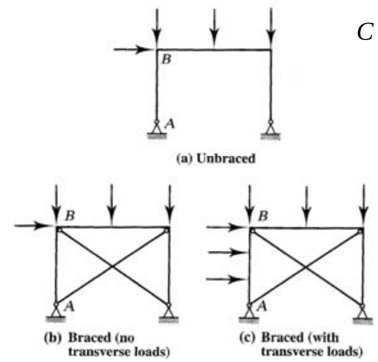
Reverse-curvature bending

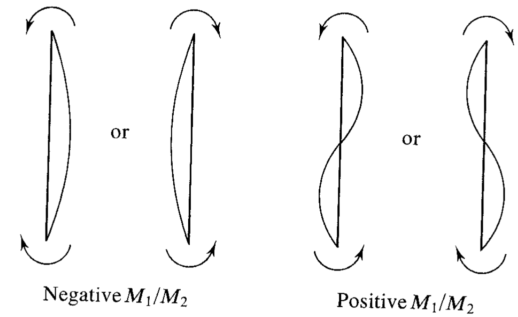


## Evaluation of $C_m$

$$C_m = 0.6 - 0.4(M_1/M_2)$$ AISC Eq. C2-4

$$C_m = 1 + \Psi(\alpha P_r/P_{e1})$$ AISC Eq. C-C2-2




(Segui Examples 6.3 & 6.5)

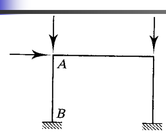
Case of no transverse loads

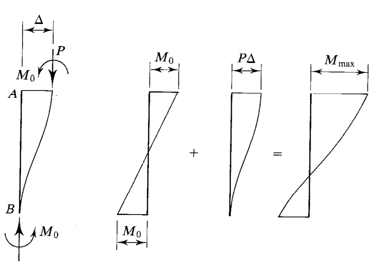


## Unbraced Frames

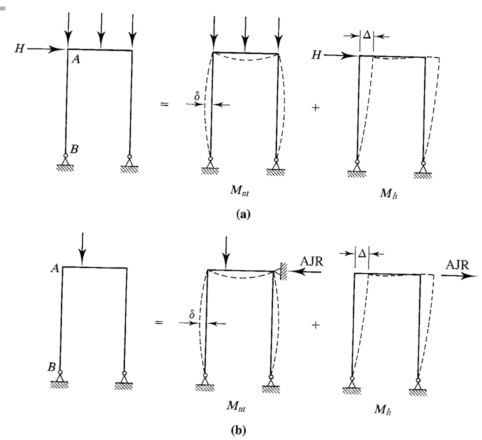
$$B_2 = \frac{1}{1 - \alpha \Sigma P_{nt} / \Sigma P_{e2}} \geq 1$$

AISC Eq. C2-3





Unbraced frames



Superposition of braced & unbraced frames

(Segui Example 6.6)



## AISC Chapter C – Stability Analysis and Design

- 2nd-order flexural strength $M_r$

$$M_r = B_1 M_{nt} + B_2 M_{lt} \qquad \text{(C2-1a)}$$

- 2nd-order axial strength $P_r$

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{(C2-1b)}$$

where

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1 \qquad \text{(C2-2)}$$

$$B_2 = \frac{1}{1 - \alpha \Sigma P_{nt} / \Sigma P_{e2}} \geq 1 \qquad \text{(C2-3)}$$

## AISC Chapter C – Stability Analysis and Design (cont.)

- $B_1$ is an amplifier to account for second order effects caused by displacement between brace points (P-$\delta$)
- $B_2$ is an amplifier to account for second order effects caused by displacements of braced points (P-$\Delta$)
- If $B_1 \leq 1.05$, it is conservative that $M_r = B_2(M_{nt} + M_{lt})$



## AISC Chapter C – Stability Analysis and Design (cont.)

- $C_m$ is a coefficient assuming no lateral translation of frame (no transverse loading)

$$C_m = 0.6 - 0.4(M_1/M_2) \quad \text{(C2-4)}$$

- $P_{e1}$ is the elastic critical buckling resistance with zero sidesway

$$P_{e1} = \frac{\Pi^2 EI}{(K_1 L)^2} \quad \text{(C2-5)}$$

- $\Sigma P_{e2}$ is the elastic critical buckling resistance for the story
  - For moment frames $\Sigma P_{e2} = \Sigma \frac{\Pi^2 EI}{(K_2 L)^2}$ (C2-6a)
  - For all types $\Sigma P_{e2} = R_M \frac{\Sigma HL}{\Delta_H}$ (C2-6b)



## $C_m$: coefficient assuming no lateral translation of the frame

- For beam-columns not subject to transverse loading – Equation (C2-5)
- For beam-columns subject to transverse

For beam-columns with transverse loadings, the second-order moment can be approximated by

$$C_m = 1 + \psi\left(\frac{\alpha P_r}{P_{e1}}\right) \quad \text{(C-C2-2)}$$

for simply supported members

where

$\psi = \frac{\pi^2 \delta_o EI}{M_o L^2} - 1$

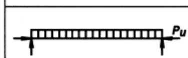
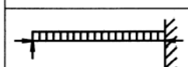
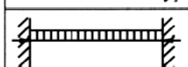
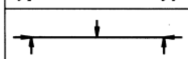
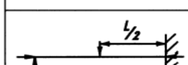
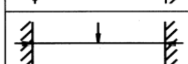
$\delta_o$ = maximum deflection due to transverse loading, in. (mm)

$M_o$ = maximum first-order moment within the member due to the transverse loading, kip-in. (N-mm)

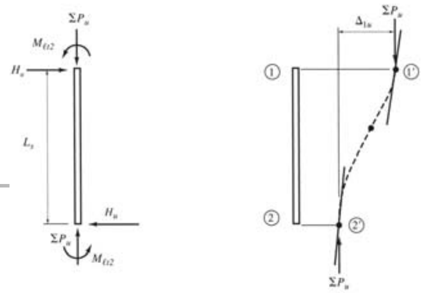
$\alpha$ = 1.0 (LRFD) or 1.6 (ASD)

## $C_m$: coefficient assuming no lateral translation of the frame (cont.)

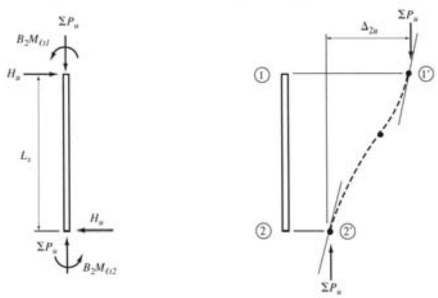
**TABLE C-C2.1**
**Amplification Factors $\psi$ and $C_m$**

| Case | $\psi$ | $C_m$ |
|---|---|---|
|  | 0 | 1.0 |
|  | –0.4 | $1 - 0.4\frac{P_u}{P_{e1}}$ |
|  | –0.4 | $1 - 0.4\frac{P_u}{P_{e1}}$ |
|  | –0.2 | $1 - 0.2\frac{P_u}{P_{e1}}$ |
|  | –0.3 | $1 - 0.3\frac{P_u}{P_{e1}}$ |
|  | –0.2 | $1 - 0.2\frac{P_u}{P_{e1}}$ |

## Summation of forces acting on all columns in one story of a multistory building frame




(a) First-order analysis: For equilibrium, $M_{lt1}+M_{lt2}=H_u L_s$




(b) Second-order analysis: For equilibrium, $B_2(M_{lt1}+M_{lt2}) = H_u L_s + \Sigma P_u \Delta_{2u}$



TABLE 12.3.1 Suggested Values for $C_m$ for Situations with No Joint Translation[a]

| Case | | $C_m$ (positive) moment) | $C_m$ (negative moment) | Primary Bending Moment |
|---|---|---|---|---|
| 1 | | $1 + 0.2\alpha^{\dagger}$ | — | $M_m$ |
| 2 | | 1.0 | — | $M_m$ |
| 3 | | $1 - 0.2\alpha$ | — | $M_m$ |
| 4 | | $1 - 0.3\alpha$ | $1 - 0.4\alpha$ | $M_m$ |



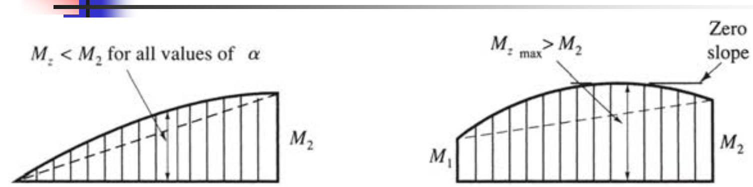
| | | | | |
|---|---|---|---|---|
| 4 | | $1 - 0.3\alpha$ | $1 - 0.4\alpha$ | $M_m$ |
| 5 | | $1 - 0.4\alpha$ | $1 - 0.4\alpha$ | $M_m$ |
| 6 | | $1 - 0.4\alpha$ | $1 - 0.3\alpha$ | $M_m$ |
| 7 | | $1 - 0.6\alpha$ | $1 - 0.2\alpha$ | $M_m$ |
| 8 | $M_A \neq M_B$ | Eq. (12.3.8) | not available | $M_m$ |

[a]Adapted from AISC Commentary Table C-C2.1

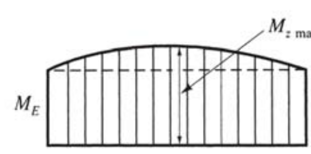
$^{\dagger}\alpha = \dfrac{P_u}{P_e} = \dfrac{P_u}{\pi^2 E/(KL/r)^2}$



## Primary plus secondary bending moment by end moments




(a) Maximum moment at ends (b) Maximum moment *not* at ends




(c) Equivalent uniform moment with maximum magnified moment at midspan

Braced frame - (Segui Example 6.8)

Unbraced frame - (Segui Example 6.10)