ENCE 455
Design of Steel Structures

III. Compression Members

C. C. Fu, Ph.D., P.E.
Civil and Environmental Engineering Department
University of Maryland

Compression Members

Following subjects are covered:

- Introduction
- Column theory
- Width/thickness limit
- Column design per AISC
- Effective length

Reading:

- Chapters 4 of Segui
- AISC Steel Manual Specification Chapters B (Design Requirements) and E (Design Members for Compression)

Introduction

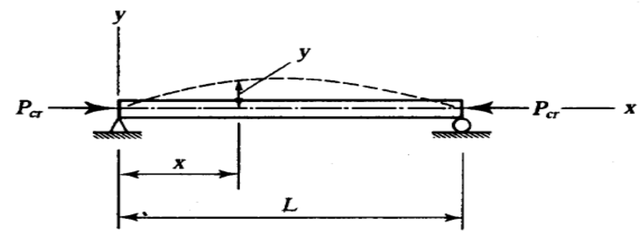
- Compression members are structural elements that are subjected only to compression forces, that is, loads are applied along a longitudinal axis through the centroid of the cross-section.
- In this idealized case, the axial stress f is calculated as

$$f = P/A$$

- Note that the ideal state is never realized in practice and some eccentricity of load is inevitable. Unless the moment is negligible, the member should be termed a beam-column and not a column, where beam columns will be addressed later.

Compression Members (cont.)

- If the axial load *P* is applied slowly, it will ultimately become large enough to cause the member to become unstable and assume the shape shown by the dashed line.
- The member has then buckled and the corresponding load is termed the critical buckling load (also termed the Euler buckling load).



Compression Members (cont.)

- The differential equation giving the deflected shape of an elastic member subject to bending is

M = P y

$$\frac{d^2y}{dz^2} + \frac{P}{EI}y = 0 \qquad (4.2)$$

where *x* is a location along the longitudinal axis of the member, *y* is the deflection of the axis at that point, *M (= P y)* is the bending moment at that point, and other terms have been defined previously.

5

Compression Members (cont.)

- The latter equation is a linear, second-order ordinary differential equation with the solution

  y=Acos(cx) + Bsin(cx)

  where *A* and *B* are constants and $c^2=P_{cr}/EI$.
- The constants are evaluated by applying the boundary conditions *y(0)=0* and *y(L)=0*. This yields *A=0* [BC 1] and *0=B sin(cL)* [BC 2].
- For a non- trivial solution (the trivial solution is *B=0*), *sin(cL)=0*, or $cL = 0, \pi, 2\pi, 4\pi, \ldots = n\pi$ and

$$P = \frac{n^2\pi^2 EI}{L^2}$$

6

Compression Members (cont.)

- Different values of n correspond to different buckling modes. A value of n=0 gives the trivial case of no load; n=1 represents the first mode, n=2 represents the second mode, etc.
- For the case of n = 1, the lowest non-trivial value of the buckling load is

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (4.3)$$

the radius of gyration *r* can be written as $I=A_g r^2$

- Then the critical buckling stress can be re-written as

$$F_{cr} = \frac{P_{cr}}{A_g} = \frac{\pi^2 E}{(L/r)^2} \qquad (4.4)$$

where *L/r* is the slenderness ratio.

7

Compression Members (cont.)

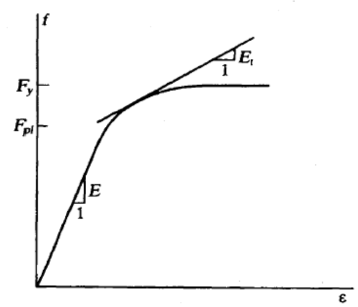
- The above equations for the critical buckling load (Euler buckling load) were derived assuming
  - A perfectly straight column
  - Axial load with no eccentricity
  - Column pinned at both ends
- If the column is not straight (initially crooked), bending moments will develop in the column. Similarly, if the axial load is applied eccentric to the centroid, bending moments will develop.
- The third assumption is a serious limitation and other boundary conditions will give rise to different critical loads. As noted earlier, the bending moment will generally be a function of *z* (and not *y* alone), resulting in a non-homogeneous differential equation.

8

Compression Members (cont.)

- The above equation does not give reliable results for stocky columns (say L/r <40) for which the critical buckling stress exceeds the proportional limit. The reason is that the relationship between stress and strain is not linear.

- For stresses between the proportional limit and the yield stress, a tangent modulus E_t is used, which is defined as the slope of the stress–strain curve for values of f between these two limits.



Compression Members (cont.)

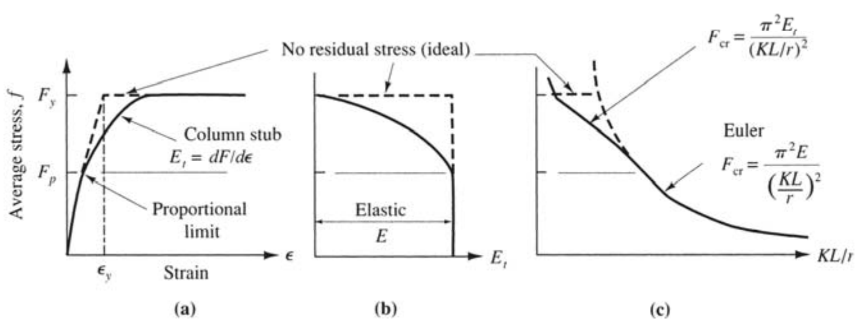
- Such a curve is seen from tests of stocky columns and is due primarily to residual stresses.
- In the transition region $F_{pl} < f \le F_y$, the critical buckling stress can be written as

$$F_{cr} = \frac{P_t}{A_g} = \frac{\pi^2 E_t}{(KL/r)^2} \qquad (4.5)$$

- But this is not particularly useful because the tangent modulus E_t is strain dependent. Accordingly, most design specifications contain empirical formulae for inelastic columns.

Compression Members (cont.)

- The critical buckling stress is often plotted as a function of slenderness as shown in the figure below. This curve is called a Column Strength Curve. From this figure it can be seen that the tangent modulus curve is tangent to the Euler curve at the point corresponding to the proportional limit.



Column Design per AISC

- The basic requirements for compression members are covered in Chapter E of the AISC Steel Manual. The basic form of the relationship is

  $P_u \le \varphi_c P_n = \varphi_c(A_g F_{cr})$ (AISC E3-1)

  where φ_c is the resistance factor for compression members (=0.9) and
- F_{cr} is the critical buckling stress (inelastic or elastic) and F_e is the elastic buckling stress

$$F_e = F_{cr} = \frac{\pi^2 E}{(KL/r)^2} \qquad \text{(AISC E3-4)}$$

Column Design per AISC (cont.)

- The nominal strength P_n of rolled compression members (AISC-E3) is given by

  $P_n = A_g F_{cr}$

  - For inelastic columns $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{QF_y}}$ or $F_e \ge 0.44QF_y$

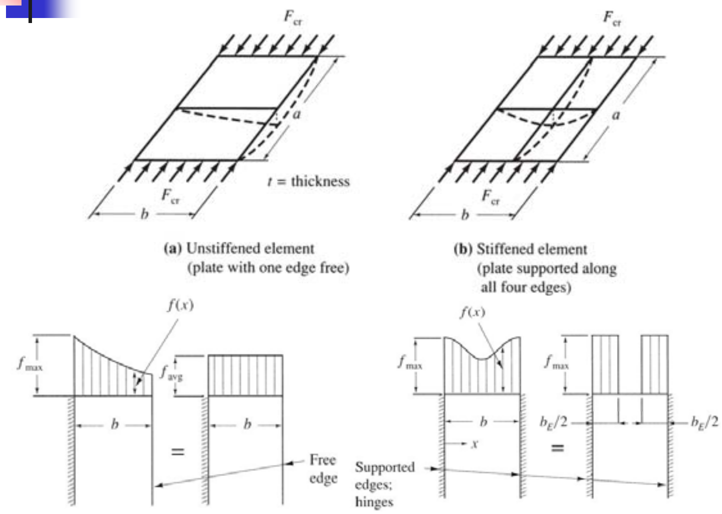
$$F_{cr} = \left(0.658^{QF_y/F_e}\right)QF_y \qquad \text{(AISC E3-2 \& E7-2)}$$

  - For elastic columns $\frac{KL}{r} > 4.71\sqrt{\frac{E}{QF_y}}$ or $F_e < 0.44QF_y$

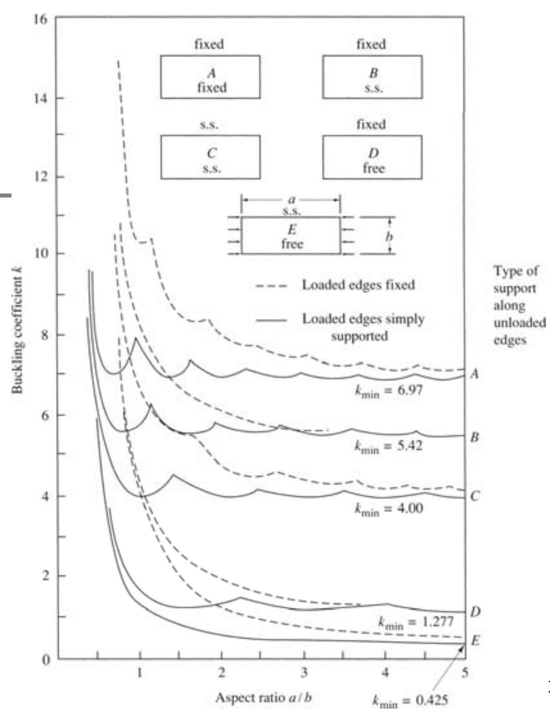
$$F_{cr} = 0.877F_e \qquad \text{(AISC E3-3 \& E7-3)}$$

- Q =1 for majority of rolled H-shaped section (Standard W, S, and M shapes); Others are covered later (Segui Example 4.2 for Q=1)

Stability of Plate



Stability of Plate (cont.)



Column Design per AISC (cont.)

Flange and web compactness

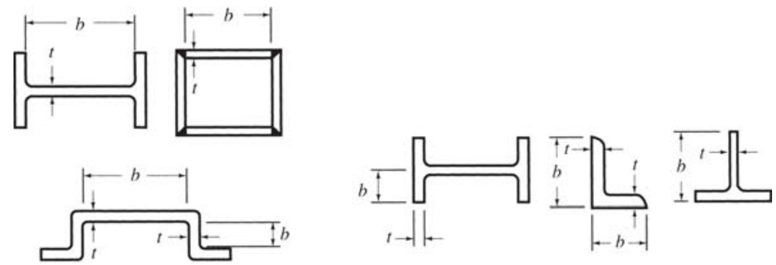
- For the strength associated with a buckling mode to develop, local buckling of elements of the cross section must be prevented. If local buckling (flange or web) occurs,
  - The cross-section is no longer fully effective.
  - Compressive strengths given by F_{cr} must be reduced
- Section B4 of the Steel Manual provides limiting values of width-thickness ratios (denoted λ_r) where shapes are classified as
  - Compact
  - Noncompact
  - Slender

Column Design per AISC (cont.)

- AISC writes that if exceeds a threshold value λ_r, the shape is considered slender and the potential for local buckling must be addressed.
- Two types of elements must be considered
  - Unstiffened elements - Unsupported along one edge parallel to the direction of load
    (AISC Table B4.1, p 16.1-16)
  - Stiffened elements - Supported along both edges parallel to the load
    (AISC Table B4.1, p 16.1-17)

Column Design per AISC (cont.)

The figure on the following page presents compression member limits (λ_r) for different cross-section shapes that have traditionally been used for design.

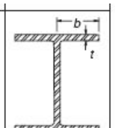
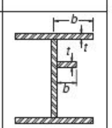
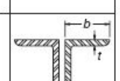


(a) Elements supported along two edges (stiffened elements)

(b) Elements supported along one edge (unstiffened elements)

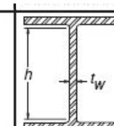
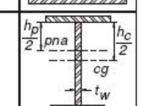
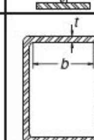
Column Design per AISC (cont.)

For unstiffened elements –

Unstiffened Elements						
	3	Uniform compression in flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections; outstanding legs of pairs of angles in continuous contact and flanges of channels	b/t	NA	$0.56\sqrt{E/F_y}$	
	4	Uniform compression in flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections	b/t	NA	$0.64\sqrt{k_cE/F_y}$ [a]	
	5	Uniform compression in legs of single angles, legs of double angles with separators, and all other unstiffened elements	b/t	NA	$0.45\sqrt{E/F_y}$	

Column Design per AISC (cont.)

For stiffened elements -

Stiffened Elements						
	10	Uniform compression in webs of doubly symmetric I-shaped sections	h/t_w	NA	$1.49\sqrt{E/F_y}$	
	11	Flexure in webs of singly-symmetric I-shaped sections	h_c/t_w	$\dfrac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{M_y}-0.09\right)^2} \le \lambda_r$	$5.70\sqrt{E/F_y}$	
	12	Uniform compression in flanges of rectangular box and hollow structural sections of uniform thickness subject to bending or compression; flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.12\sqrt{E/F_y}$	$1.40\sqrt{E/F_y}$	

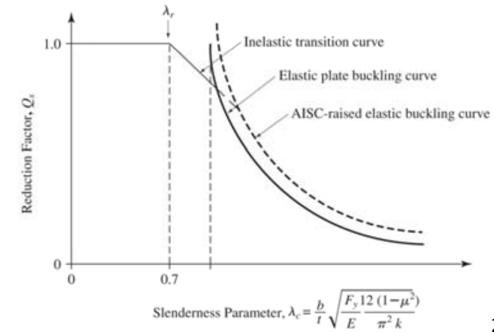
Column Design per AISC (cont.)

- $\lambda > \lambda_r$ in an element of a member, the design strength of that member must be reduced because of local buckling. The general procedure for this case is as follows:
- Compute a reduction factor Q per E7.1 (unstiffened compression elements Q_s) or E7.2 (stiffened compression elements Q_a).

Reduction Factor Q

- Unstiffened compression elements: Compute a reduction factor Q_s per E7.1
- Stiffened compression elements: Compute a reduction factor Q_a per E7.2

Unstiffened compression element



Reduction Factor Q (cont.)

- AISC-E7.1 (Stiffened elements)
  - For other uniformly compressed elements:

$$b_E = 1.92t\sqrt{\frac{E}{f}}\left[1.0 - \frac{0.38}{(b/t)}\sqrt{\frac{E}{f}}\right] \le b \quad \text{(AISC E7-18)}$$

  - $f = P_u/A_g = \varphi_c Q_s F_{cr,column}$
  - $Q_a = A_{eff}/A_{gross} = b_E t/(bt)$
    where $A_{eff} = A_{gross} - \Sigma(b-b_E)t$

Reduction Factor Q (cont.)

Design Properties

- In computing the nominal strength, the following rules apply in accordance with AISC-E7
  - For axial compression
    1. Use gross area A_g for $P_n = F_{cr}A_g$
    2. Use gross area to compute radius of gyration r for KL/r
  - For flexure:
    1. Use reduced section properties for beams with flanges containing stiffened elements

(cont…)

(Segui Example 4.4 with reduction factor Q to check local buckling)

Reduction Factor Q (cont.)

Design Properties (cont.)

- Since the strengths of beams do not include Q factors relating to thin compression elements, it is appropriate to use section properties based on effective area
  - For beam columns:
    1. Use gross area for P_n
    2. Use reduced section properties for flexure involving stiffened compression elements for M_{nx} and M_{ny}
    3. Use Q_a and Q_s for determining P_n
    4. For F_{cr} based on lateral-torsional buckling of beams as discussed later in Beams, the maximum value of F_{cr} is Q_sF_{cr} when unstiffened compression elements are involved.

TABLE 6.8.1 Axial Compression—AISC Specification References

Topic	Specification sections AISC [1.13]
Local buckling limits for “noncompact” sections	B4
Local buckling limits for “compact” sections	B4
Slenderness limits	E2
Moment resisting frame, definition	C1.3a
Unbraced frame, definition	C1.3b
Effective length factors	C2
Column formulas, basic	E3
Torsional and flexural torsional buckling	E4
Single angles	E5
Built-up members	E6
Slender compression elements	E7
Alignment chart	Commentary C2

AISC of Rolled Shape Columns

The general design procedure is:

1. Computer the factor service load P_u using all appropriate load combinations
2. Assume a critical stress F_{cr} based on assumed KL/r
3. Computer the gross area A_g required from $P_u/(\varphi_c F_{cr})$
4. Select a section. Note that the width/thickness λ_r limitations of AISC Table B4.1 to prevent local buckling must be satisfied.

   (cont…)

(Segui Example 4.5 based on AISC Table 4-22 and Example 4.6 based on AISC Table 4-1)

AISC of Rolled Shape Columns (cont.)

5. Based on the larger of $(KL/r)_x$ or $(KL/r)_y$ for the section selected, compute the critical stress F_{cr}.
6. Computer the design strength $\varphi_c P_n = \varphi_c F_{cr} A_g$ for the section.
7. Compare $\varphi_c P_n$ with P_u. When the strength provided does not exceed the strength required by more than a few percent, the design would be acceptable. Otherwise repeat Steps 2 through 7.

(Segui Examples 4.10 & 4.11 for rolled shape)

Column Design per AISC (cont.)

Tables for design of compression members -

- Tables 4.2 through 4.17 in Part 4 of the AISC Steel Manual present design strengths in axial compression for columns with specific yield strengths, for example, 50 ksi for W shapes. Data are provided for slenderness ratios of up to 200.
- Sample data are provided on the following page for some W14 shapes

Column Design per AISC (cont.)

W14 samples

(AISC LRFD p 4-21)

Table 4-2 (cont.)
W-Shapes
Design Strength in Axial Compression, $\phi_c P_n$ kips

$F_y = 50$ ksi
$\phi_c P_n = 0.85 F_{cr} A_g$

Shape	W14× 311*	283*	257*	233*	211	193	176	159	145	132
Effective length *KL* (ft) with respect to least radius of gyration r_y										
0	3880	3540	3210	2910	2640	2410	2200	1980	1810	1650
11	3610	3290	2980	2700	2440	2230	2030	1830	1670	1510
12	3560	3240	2940	2660	2400	2200	2000	1810	1650	1480
13	3510	3200	2890	2620	2370	2170	1970	1780	1620	1450
14	3460	3140	2850	2570	2330	2130	1940	1740	1590	1430
15	3400	3090	2800	2530	2280	2090	1900	1710	1560	1390
16	3330	3030	2740	2480	2240	2050	1860	1680	1530	1360
17	3270	2970	2690	2430	2190	2010	1820	1640	1500	1330
18	3200	2910	2630	2380	2140	1960	1780	1600	1460	1300
19	3130	2850	2570	2320	2090	1910	1740	1570	1430	1260
20	3060	2780	2510	2270	2040	1870	1700	1530	1390	1220
22	2910	2640	2380	2150	1940	1770	1610	1440	1320	1150
24	2750	2500	2250	2030	1830	1670	1510	1360	1240	1070
26	2590	2350	2120	1910	1710	1560	1420	1270	1160	997
28	2430	2200	1980	1780	1600	1460	1320	1190	1080	920
30	2270	2050	1840	1660	1490	1350	1220	1100	996	844
32	2110	1900	1710	1530	1370	1250	1130	1010	919	769
34	1950	1760	1570	1410	1260	1150	1040	928	842	697
36	1790	1620	1440	1290	1160	1050	946	846	767	627
38	1640	1480	1320	1180	1050	965	859	767	694	563
40	1490	1340	1190	1070	951	863	775	692	626	508
42	1350	1220	1080	967	863	783	703	628	568	461
44	1230	1110	987	881	786	713	641	572	518	420
46	1130	1010	903	806	719	652	586	523	474	384
48	1040	932	829	741	660	599	538	481	435	353
50	956	858	764	682	609	552	496	443	401	325
Properties										
P_{wo}, kips	1010	861	735	621	529	454	396	333	287	263
P_{wi}, kips/in.	70.5	64.5	59.0	53.5	49.0	44.5	41.5	37.3	34.0	32.3
P_{wb}, kips	6390	4900	3730	2780	2150	1610	1310	944	716	611
P_{fb}, kips	1440	1210	1000	832	684	583	493	398	334	298
L_p, ft	14.8	14.7	14.6	14.5	14.4	14.3	14.2	14.1	14.1	13.3
L_r, ft	110	100	91.6	83.4	76	70.1	64.5	58.9	54.7	49.6

Effective Length

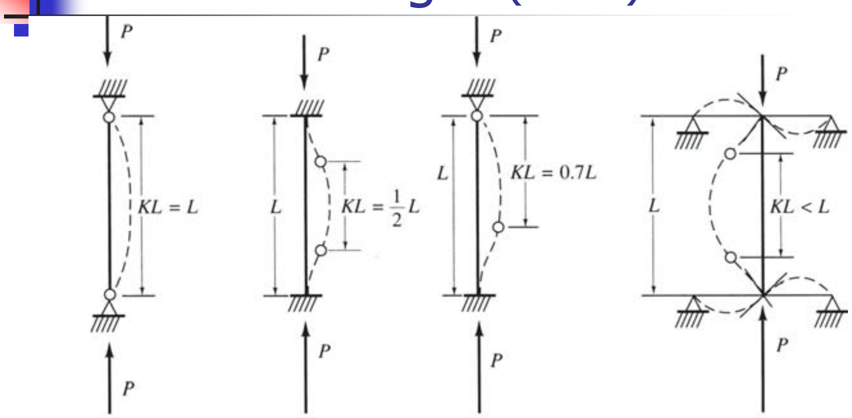
- Consider the column that is pinned at one end (*y(0)=y″(0)=0*) and fixed against translation and rotation at the other end (*y(0)=y′(0)=0*). The critical buckling load is:

$$P_{cr} = \frac{\pi^2 EI}{(0.7L)^2}$$

- Another case is fixed at one end (*y(0)=y′(0)=0*) and free at the other end. The critical buckling load is:

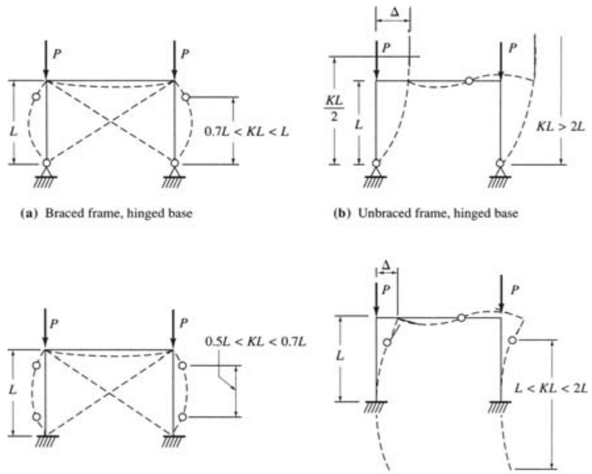
$$P_{cr} = \frac{\pi^2 EI}{(2.0L)^2}$$

Effective Length (cont.)



(a) End rotations unrestrained **(b)** End rotations fully restrained **(c)** One end restrained, other unrestrained **(d)** Partially restrained at each end

Effective Length (cont.)



(a) Braced frame, hinged base

(b) Unbraced frame, hinged base

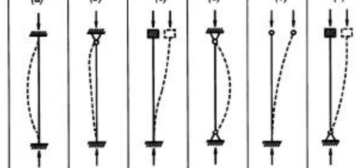
(c) Braced frame, fixed base

(d) Unbraced frame, fixed base

Effective Length (cont.)

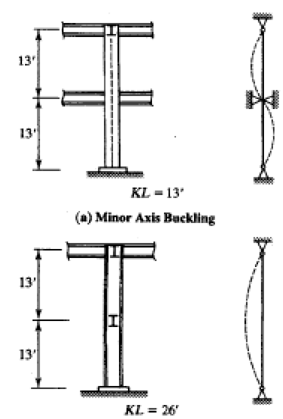
- The AISC Steel Manual presents a table to aid in the calculation of effective length. Theoretical and design values are recommended. The conservative design values should generally be used unless the proposed end conditions truly match the theoretical conditions.

TABLE C-C2.2
Approximate Values of Effective Length Factor, K

Buckled shape of column is shown by dashed line.	(a)	(b)	(c)	(d)	(e)	(f)
						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	Rotation fixed and translation fixed; Rotation free and translation fixed; Rotation fixed and translation free; Rotation free and translation free					

Effective Length (cont.)

- The AISC table presented earlier presents values for the design load based on a slenderness ratio calculated using the minimum radius of gyration, r_y . Consider now the figure shown.



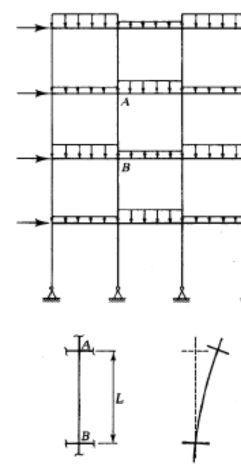
(a) Minor Axis Buckling

(b) Major Axis Buckling

Effective Length (cont.)

For columns in moment-resisting frames, the tabulated values of K presented on Table C-C2.1 of AISC Steel Manual will not suffice for design. Consider the moment-frame shown that is permitted to sway.

- Columns neither pinned not fixed.
- Columns permitted to sway.
- Columns restrained by members framing into the joint at each end of the column

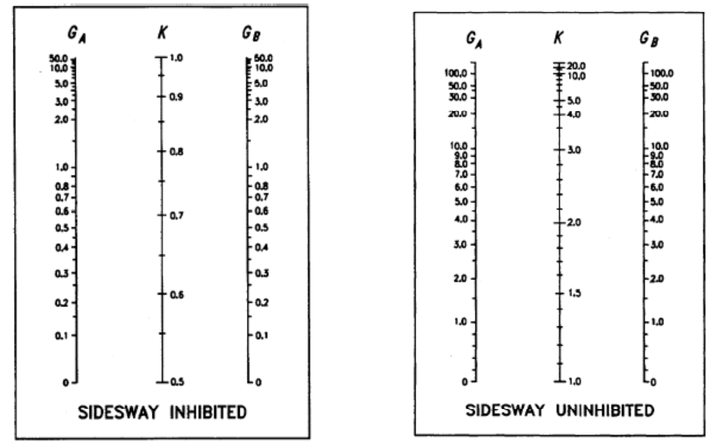


Effective Length (cont.)

The effective length factor for a column along a selected axis can be calculated using simple formulae and a nomograph. The procedure is as follows:

- Compute a value of G, defined below, for each end of the column, and denote the values as G_A and G_B , respectively $G = \frac{\Sigma(EI/L)_{col}}{\Sigma(EI/L)_{beam}}$
- Use the nomograph provided by AISC (and reproduced on the following pages). Interpolate between the calculated values of G_A and G_B to determine K

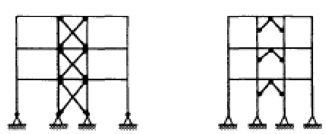
Effective Length (cont.)



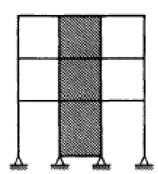
AISC specifies G = 10 for a pinned support and G = 1.0 for a fixed support.

Effective Length (cont.)

- The distinction between braced (sidesway inhibited) and unbraced (sidesway inhibited) frames is important, as evinced by difference between the values of K calculated above.
- What are bracing elements?



(a) Diagonal bracing



(b) Shear Walls (masonry, reinforced concrete, or steel plate)

Effective Length (cont.)

- Above presentation assumed that all behavior in the frame was elastic. If the column buckles inelastically ($\lambda_c \leq 1.5$), then the effective length factor calculated from the alignment chart will be conservative. One simple strategy is to adjust each value of G using a stiffness reduction factor (SRF),

$$G_{inelastic} = \frac{\Sigma(E_t I/L)_{col}}{\Sigma(EI/L)_{beam}} = G_{elastic}[\tau_a]$$

$$\tau_a = \frac{E_t}{E} = \frac{F_{cr,inelastic}}{F_{cr,elastic}} \quad (4.13)$$

- Table 4-21 of the AISC Steel Manual, presents values for the SRF (AISC called τ) for various values of F_y and P_u/A_g. (Segui Example 4.14)