ENCE 455
Design of Steel Structures

IV. Beams

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Introduction

Following subjects are covered:

- Introduction
- Stability
- Compact shapes
- Non-compact shapes
- Shear strength
- Serviceability
- Beam bearing plates & Column base plates
- Biaxial bending

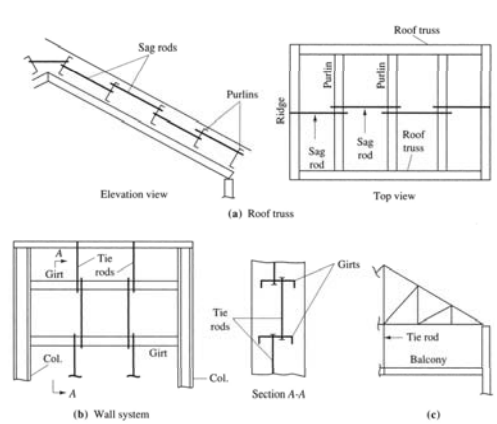
Reading:

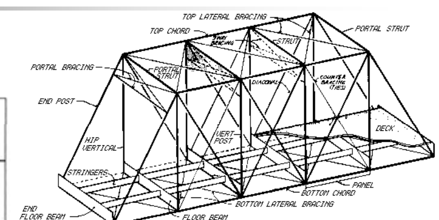
- Chapters 5 of Segui
- AISC Steel Manual Specifications Chapters B (Design Requirements), F (Beams and Other Flexural Members), and L (Serviceability Design)

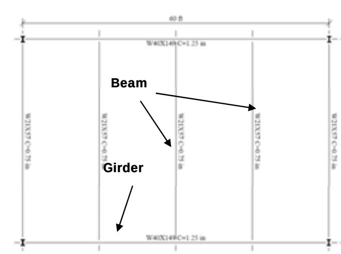
Introduction (cont.)

- Flexural members/beams are defined as members acted upon primarily by transverse loading, often gravity dead and live load effects. Thus, flexural members in a structure may also be referred to as:
  - Girders – usually the most important beams, which are frequently at wide spacing.
  - Joists – usually less important beams which are closely spaced, frequently with truss-type webs.
  - Purlins – roof beams spanning between trusses.
  - Stringers – longitudinal bridge beams spanning between floor beams.
  - Girts – horizontal wall beams serving principally to resist bending due to wind on the side of an industrial building, frequently supporting corrugated siding.
  - Lintels – members supporting a wall over window or door openings

Introduction (cont.)







based on FloorFraming Program

Stability

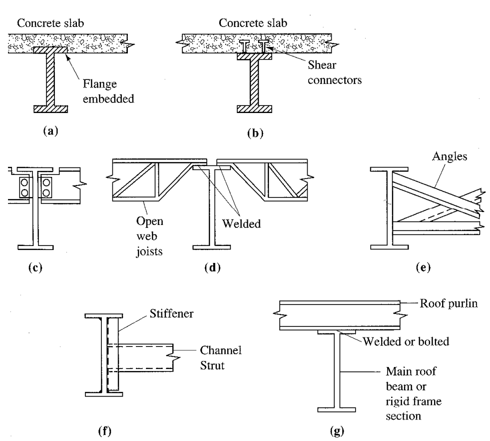
- The laterally supported beams assume that the beam is stable up to the fully plastic condition, that is, the nominal strength is equal to the plastic strength, or $M_n = M_p$
- If stability is not guaranteed, the nominal strength will be less than the plastic strength due to
  - Lateral-torsional buckling (LTB)
  - Flange and web local buckling (FLB & WLB)
- When a beam bends, one half (of a doubly symmetric beam) is in compression and, analogous to a column, will buckle.

Stability (cont.)

- Unlike a column, the compression region is restrained by a tension region (the other half of the beam) and the outward deflection of the compression region (flexural buckling) is accompanied by twisting (torsion). This form of instability is known as lateral- torsional buckling (LTB)
- LTB can be prevented by lateral bracing of the compression flange. The moment strength of the beam is thus controlled by the spacing of these lateral supports, which is termed the unbraced length.

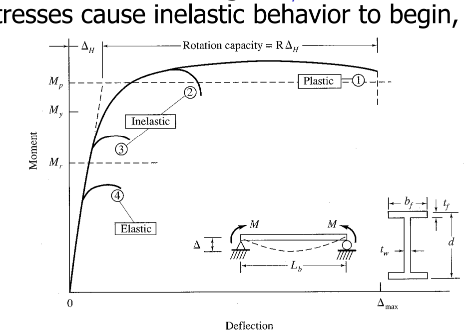
Stability (cont.)

- Flange and web local buckling (FLB and WLB, respectively) must be avoided if a beam is to develop its calculated plastic moment.



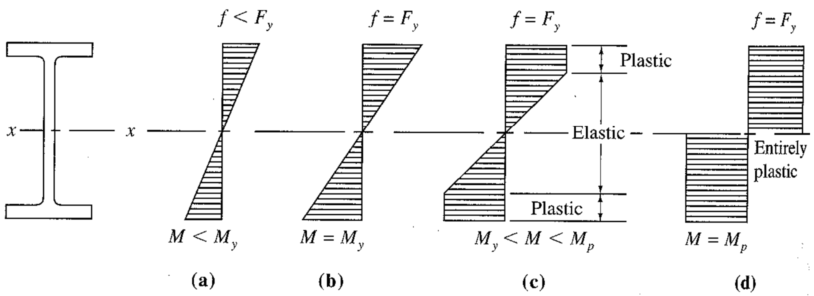
Stability (cont.)

- Four categories of behavior are shown in the figure:
  - Plastic moment strength M_p along with large deformation.
  - Inelastic behavior where plastic moment strength M_p is achieved but little rotation capacity is exhibited.
  - Inelastic behavior where the moment strength M_r, the moment above which residual stresses cause inelastic behavior to begin, is reached or exceeded.
  - Elastic behavior where moment strength M_{cr} is controlled by elastic buckling.



Compact Shapes

- The stress distribution on a typical wide-flange shape subjected to increasing bending moment is shown below



Compact Shapes (cont.)

- In the service load range the section is elastic as in (a)
- When the yield stress is reached at the extreme fiber (b), the yield moment M_y is

  $M_n = M_y = S_x F_y$
- When the condition (d) is reached, every fiber has a strain equal to or greater than $\varepsilon_y = F_y/E_s$, the plastic moment M_p is

$$M_P = F_y \int_A y dA = F_y Z$$

Where Z is called the plastic modulus

Compact Shapes (cont.)

- Note that ratio, shape factor ξ, M_p/M_y is a property of the cross-sectional shape and is independent of the material properties.

  $\xi = M_p/M_y = Z/S$
- Values of S and Z (about both x and y axes) are presented in the Steel Manual Specification for all rolled shapes.
- For W-shapes, the ratio of Z to S is in the range of 1.10 to 1.15

  (Segui Example 5.1 for S & M_y, Z & M_p)

Compact Shapes (cont.)

- The AISC strength requirement for beams:

  $\phi_b M_n \geq M_u$
  - Compact sections: $M_n = M_p = Z F_y$ (AISC F2-1)

    where for I-shaped member

    $\lambda = b_f/2t_f \leq \lambda_p$ $(=0.38\sqrt{E/F_y})$ for flanges

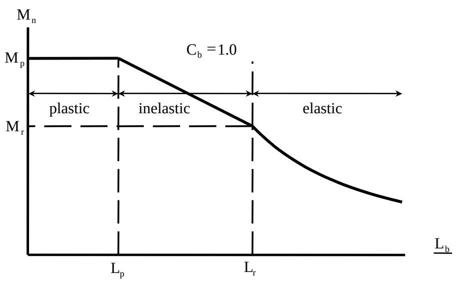
    $= h/t_w \leq \lambda_p$ $(=3.76\sqrt{E/F_y})$ for beam web

    λ_r, λ_p from Segui Tables 5.3 or AISC Table B4.1

    (Segui Example 5.3)

Bending Strength of Compact Shapes

- Nominal Flexural Strength M_n
  - plastic when $L_b \le L_p$ and $M_n = M_p$
  - inelastic when $L_p < L_b \le L_r$ and $M_p > M_n \ge M_r$
  - elastic when $L_b > L_r$ and $M_n < M_r$



Bending Strength of Compact Shapes (cont.)

- Plastic LTB (Yielding)
  - Flexural Strength $M_n = M_p = F_y Z$ (AISC F2-1)

  where Z= plastic section modulus & F_y= section yield stress
  - Limits
    - Lateral bracing limit $L_b < L_p = 1.76 r_y \sqrt{\dfrac{E}{F_y}}$ (AISC F2-5)
    - Flange and Web width/thickness limit (AISC Table B4.1)

Bending Strength of Compact Shapes (cont.)

- Inelastic LTB $L_p < L_b \le L_r$
  - Flexure Strength (straight line interpolation)

$$M_n = C_b \left[M_p - \left(M_p - M_r \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \le M_p$$

or

$$M_n = C_b \left[M_p - \left(M_p - 0.7 F_y S_x \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \le M_p \quad \text{(AISC F2-2)}$$

(Segui Example 5.4)

Bending Strength of Compact Shapes (cont.)

- Elastic LTB $L_b > L_r$
  - Flexure Strength

$$M_n = F_{cr} S_x \le M_p \quad \text{(AISC F2-3)}$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}} \right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}} \right)^2} \quad \text{(AISC F2-4)}$$

(The square root term may be conservatively taken equal to 1.0)
(c in AISC F2-8a,b for doubly symmetric I-shape, and channel, respectively)

  - Limit

$$L_r = 1.95 r_{ts} \frac{E}{0.7 F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{0.7 F_y}{E} \frac{S_x h_o}{Jc} \right)^2}} \quad \text{(AISC F2-6)}$$

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \quad \text{(AISC F2-7)}$$

Bending Strength of Compact Shapes (cont.)

- Moment Gradient Factor C_b
  - The moment gradient factor C_b accounts for the variation of moment along the beam length between bracing points. Its value is highest, $C_b=1$, when the moment diagram is uniform between adjacent bracing points.
  - When the moment diagram is not uniform

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \qquad \text{(AISC F1-1)}$$

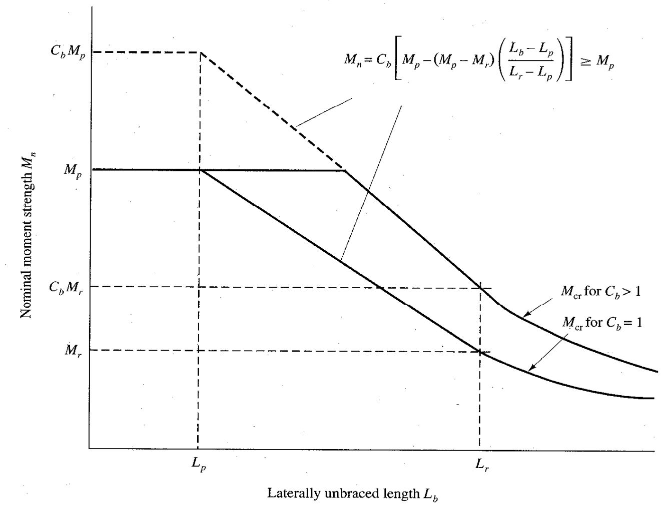
where

M_{max}= absolute value of maximum moment in unbraced length

M_A, M_B, M_C= absolute moment values at one-quarter, one-half, and three-quarter points of unbraced length

(Segui Example 5.5/Figure 5.15 & Example 5.6)

Nominal Moment Strength M_u as affected by C_b



Shear on Rolled Beams

- General Form $v = VQ/(It)$ and average form is
  $f_v = V/A_w = V/(dt_w)$
- AISC-F2
  $\phi_v V_n \geq V_u$
  where
  $\phi_v = 1.0$
  $V_n = 0.6F_{yw}A_w$ for beams without transverse stiffeners and $h/t_w \leq 2.24/\sqrt{E/F_y}$

Serviceability of Beam

- Deflection
  - AISC – Section L3: Deformations in structural members and structural system due to service loads shall not impair the serviceability of the structure
  - ASD - $\Delta_{max} = 5wL^4/(384EI)$

    As a guide in Segui Table 5.4 – Max. live load deflection

    - L/360, L/240, L/180 (roof); L/360 (floor beam)

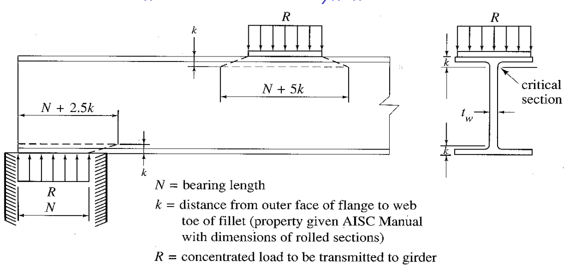
  (Segui Example 5.9)

Beam Bearing Plates

- AISC-J10.2 $\phi R_n \geq R_u$
  - Local web yielding (use R_1 & R_2 in AISC Table 9-4)
    1. Interior loads

       $R_n = (5k + N)F_{yw}t_w$ *(AISC J10-2)*
    2. End reactions

       $R_n = (2.5k + N)F_{yw}t_w$ *(AISC J10-2)*



Beam Bearing Plates (cont.)

- AISC-J10.3 (cont.)
  - Web Crippling (use R_3, R_4, R_5 & R_6 in AISC Table 9-4)
    1. Interior loads

$$R_n = 0.80t_w^2\left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$ *(AISC J10-4)*

    2. End reactions

$$R_n = 0.4t_w^2\left[1+3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$ *(AISC J10-5a) for N/d≤ 0.2*

$$R_n = 0.4t_w^2\left[1+\left(\frac{4N}{d}-0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$ *(AISC J10-5b) for N/d>0.2*

(Segui Example 5.16)

Column Base Plates

- Thornton's approach ($\varphi_c = 0.60$)

$$t \geq l\sqrt{\frac{2P_u}{0.90BNF_y}} \quad (5.18)$$

$$l = \max(m, n, \lambda n')$$

$$m = \frac{N-0.95d}{2} \qquad n = \frac{B-0.80b_f}{2}$$

$$\lambda = \frac{2\sqrt{X}}{1+\sqrt{1-x}} \leq 1 \qquad X = \left[\frac{4db_f}{(d+b_f)^2}\right]\frac{P_u}{\phi_c P_p}$$

$$n' = \frac{1}{4}\sqrt{db_f} \qquad P_p = 0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} \leq 1.7f_c'A_1$$

(Segui Example 5.17)

Biaxial Bending

- AISC-H2

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \leq 1.0 \quad (5.22)$$

  - Compact shape

$$M_{ny} = M_{py} = F_y Z_y \leq 1.6F_y S_y \quad \text{(AISCF6.1)}$$

  - Non-compact shape

$$M_{ny} = M_{py} - \left(M_{py} - 0.7F_y S_y\right)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \quad \text{(AISCF6.2)}$$

(Segui Examples 5.18 & 5.19)